School Assignment and Labor-Market Skill Formation

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May 9, 2026

Abstract

This paper studies how post-school skill formation in the labor market shapes optimal student assignment in schools. In a simple dynamic assignment model, positive assortative matching in schools is not necessarily efficient when worker skills remain fixed after schooling, and this result can persist when skills evolve autonomously. By contrast, when worker skill evolves complementarily with firm productivity, positive assortative matching in schools can be optimal under plausible conditions. The results show that optimal education policy depends not only on peer complementarities within schools, but also on worker–firm complementarities in subsequent skill formation.

JEL Classification: C78, D63, I24, I28, J24
Keywords: Assortative matching, Human capital, Skill formation, Education policy, Labor markets

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1 Introduction

A central question in assignment problems is whether high-type agents should be matched with other high-type agents. The classical assignment literature shows that positive assortative matching (PAM) is efficient when the relevant production or surplus function is supermodular (Becker, 1973; Koopmans and Beckmann, 1957; Shapley and Shubik, 1971). This insight has shaped how economists think about matching between firms and workers, husbands and wives, students and schools, and students and teachers. Much of the subsequent literature has therefore studied sorting and complementarities within a given market.[1]

This paper emphasizes that the optimal assignment rule in schools cannot be determined by the school market alone; the downstream labor market also matters. As Durlauf (2026) shows, once school assignments shape the distribution of skills entering the labor market in a dynamic setting, PAM in schools need not be efficient, unlike in the static case. Importantly, this occurs even when both school skill production and labor-market production are supermodular, provided that worker skill is fixed in the labor market.[2] However, worker skill or human capital may change over time through human capital accumulation or learning by doing, and this process may depend on the worker–firm match.

This observation motivates the central question of this paper: how does post-school skill formation in the labor market affect the optimal assignment of students in schools? To answer this question, I develop a simple dynamic assignment model with students, schools, and firms. As in Durlauf (2026), students are first matched with schools, and these matches determine the distribution of post-school skills. Workers are then matched with firms, and output is produced using worker skill and firm productivity. A social planner chooses assignments to maximize aggregate output.

The model delivers three results. First, the Durlauf-type non-PAM result continues to

[1]For labor-market applications, see Abowd et al. (1999), Shimer and Smith (2000), and Eeckhout and Kircher (2011).

[2]To be clear, Durlauf (2026) does not argue that school assignments should be negatively assortative. Rather, the paper shows that dynamic assignment problems require caution because optimal rules may depend on complex and uncertain downstream interactions.

hold when worker skill remains fixed during the labor-market stage, regardless of the number of labor-market periods. Thus, the breakdown of school PAM is not driven by dynamics per se, but by the fact that school assignments shape the skill distribution entering the labor market. Second, when worker skill evolves complementarily with firm productivity, school PAM can be optimal under plausible conditions. Third, when skill growth is independent of matched firm productivity, the non-PAM result can persist even if worker skill grows over time. These results show that optimal school assignment depends not only on peer complementarities within schools, but also on the nature of post-school skill formation in the labor market.

This paper contributes to the literature by linking assignment problems across two stages of the life cycle. Much of the matching literature studies assortative matching within a single market or assignment stage. Building on Durlauf (2026), I study how the interaction between the education stage and the subsequent labor market shapes optimal assignment, and what sustains or breaks PAM in schools. The key insight is that the efficient school assignment depends on the downstream skill-formation technology: the same school production function can imply different optimal school assignments depending on whether post-school skills are fixed, evolve autonomously, or evolve through worker–firm complementarities. This finding adds a labor-market dimension to education policy, as it implies that optimal student assignment should account for how workers' human capital evolves after schooling.

This perspective is relevant for education policies such as tracking, ability grouping, teacher assignment, and the allocation of students across schools. These policies are often evaluated using contemporaneous peer effects or school-level production. The results below show that their efficiency also depends on how the resulting skill distribution is transformed in the labor market. A policy that appears inefficient when evaluated only by contemporaneous school outcomes may become efficient once its effects on later labor-market skill formation are taken into account, and vice versa.

Related literature. This paper is related to the theoretical literature on assortative matching and assignment. The static benchmark goes back to Becker (1973): when the payoff function exhibits increasing differences, positive assortative matching is efficient. Durlauf and Seshadri (2003) extend this logic to group-assignment environments, and Anderson (2015) studies a dynamic generalization of Becker's result. The closest paper is Durlauf (2026); I extend its assignment logic by allowing post-school skill formation to depend on worker–firm matches.

The paper also relates to empirical work on worker–firm sorting, skill returns, and workplace learning. Recent matched employer–employee evidence suggests that firms differ not only in average wage premia, but also in how worker skills are rewarded and accumulated. Böhm et al. (2025) document substantial firm heterogeneity in returns to cognitive and noncognitive skills and show that workers with high endowments of a skill tend to sort into firms with high returns to that skill. Lise and Postel-Vinay (2020) estimate a model of multidimensional skills in which workers accumulate skills when those skills are used in jobs with corresponding skill requirements. Relatedly, Jarosch et al. (2019) provide evidence that exposure to higher-paid coworkers is associated with stronger subsequent wage growth, consistent with workplace learning. These findings do not directly estimate the scalar skill-transition function used here, but they motivate the assumption that post-school skill formation may depend on the firm or job to which a worker is assigned.

The paper proceeds as follows. Section 2 introduces a two-period benchmark that refines the argument in Durlauf (2026). Section 3 develops a three-period model to study how labor-market skill formation affects optimal school assignment. Section 4 concludes.

2 A Two-Period Benchmark with Fixed Worker Skill

This section presents the baseline dynamic assignment problem. The main point is simple. Even if school skill production is supermodular, positive assortative matching in schools need not be optimal once school assignments affect the distribution of skills entering a subsequent labor market. This result follows the logic of Durlauf (2026); the formulation below makes explicit that the conclusion obtains when post-school skills are fixed in the labor market.

2.1 Environment

There are two periods. In the first period, a planner assigns students with initial skill $x \in X \subset \mathbb{R}$ to schools with teaching ability $z \in Z \subset \mathbb{R}$. Their distributions are Γ and Λ. If student x is assigned to school z, post-school skill is

$$x' = g(x, z).$$

The function g is strictly increasing and strictly supermodular:

$$g(x_2, z_2) + g(x_1, z_1) > g(x_2, z_1) + g(x_1, z_2)$$

for all $x_2 > x_1$ and $z_2 > z_1$.

In the second period, workers with post-school skill x' are assigned to firms with managerial quality $y \in Y \subset \mathbb{R}$, distributed according to Φ. Output is

$$f(x', y),$$

where f is strictly increasing and strictly supermodular:

$$f(x'_2, y_2) + f(x'_1, y_1) > f(x'_2, y_1) + f(x'_1, y_2)$$

for all $x'_2 > x'_1$ and $y_2 > y_1$.

Let $\chi : X \to Z$ denote the school assignment, with $\chi_{\#}\Gamma = \Lambda$. Given χ, the induced post-school skill distribution is

$$\Gamma'_\chi = \big(g(\cdot, \chi(\cdot))\big)_{\#}\Gamma.$$

Let $\mu : X' \to Y$ denote the firm assignment, with $\mu_{\#}\Gamma'_\chi = \Phi$.

2.2 Social Planner's Problem

The planner solves the following problem:

$$\max_{\chi,\mu} \int_{X'} f(x', \mu(x'))\, d\Gamma'_\chi(x') \tag{1}$$

subject to

$$\chi_{\#}\Gamma = \Lambda, \qquad \Gamma'_\chi = \big(g(\cdot, \chi(\cdot))\big)_{\#}\Gamma, \qquad \mu_{\#}\Gamma'_\chi = \Phi.$$

Proposition 2.1 (Labor-market PAM and possible non-PAM school assignment)**.** *For any feasible school assignment χ, strict supermodularity of f implies that the optimal firm assignment is positive assortative in post-school skill x'. However, strict supermodularity of g does not imply that the optimal school assignment is positive assortative. There exist economies satisfying the assumptions above in which a non-assortative school assignment strictly dominates school PAM.*

The proof follows the pairwise-exchange argument for the labor-market assignment and a Durlauf-type counterexample. For completeness, it is provided in Section A.

3 Three-Period Assignment with Labor-Market Skill Formation

This section shows that the optimal school assignment depends not only on the presence of a downstream labor market, but also on the way worker skills evolve in that market. I extend the baseline model in Section 2 by adding one more labor-market period. The key result is a cutoff result: if worker–firm complementarities in post-school skill formation are weak, school NAM can remain optimal; if they are sufficiently strong, school PAM can be optimal.

3.1 Environment and Social Planner's Problem

The environment is identical to that in Section 2, except that the labor market lasts for two periods. Given a school assignment χ, post-school skill is

$$x' = g(x, \chi(x)),$$

and the induced post-school skill distribution is

$$\Gamma'_{\chi} = \big(g(\cdot, \chi(\cdot))\big)_{\#}\Gamma.$$

Workers are assigned to firms according to $\mu : X' \to Y$, where

$$\mu_{\#}\Gamma'_{\chi} = \Phi.$$

Firm productivity y is fixed over time, and worker–firm matches formed in the first labor-market period are maintained in the second labor-market period.[3]

Output in the first labor-market period is

$$f(x', \mu(x')).$$

Worker skill in the second labor-market period is

$$x'' = h(x', \mu(x')). \tag{2}$$

[3] This assumption is made for expositional simplicity. Allowing for rematching in the second labor-market period does not change the argument as long as the skill transition preserves the ranking of workers. In that case, the period-2 PAM assignment remains PAM after skills evolve.

Output in the second labor-market period is

$$f(x'', \mu(x')) = f(h(x', \mu(x')), \mu(x')).$$

The planner chooses the school assignment χ and the labor-market assignment μ to solve

$$\begin{aligned} \max_{\chi,\mu} \quad & \int_{X'} [f(x', \mu(x')) + f(h(x', \mu(x')), \mu(x'))] \, d\Gamma'_\chi(x') \\ \text{s.t.} \quad & \chi_\# \Gamma = \Lambda, \\ & \Gamma'_\chi = \big(g(\cdot, \chi(\cdot))\big)_\# \Gamma, \\ & \mu_\# \Gamma'_\chi = \Phi. \end{aligned} \tag{3}$$

Define the composite labor-market value of matching post-school skill x' with firm productivity y as

$$F(x', y) = f(x', y) + f(h(x', y), y). \tag{4}$$

Then the planner's problem is equivalent to

$$\max_{\chi,\mu} \int_{X'} F(x', \mu(x')) \, d\Gamma'_\chi(x')$$

subject to the same feasibility constraints. Thus the three-period problem has the same assignment structure as Section 2, but with f replaced by the composite value F.

3.2 A Cutoff Result for Worker–Firm Complementarity

I now isolate the role of worker–firm complementarities in post-school skill formation. Let the composite labor-market value be written as

$$F_\rho(x', y) = F_0(x', y) + \rho H(x', y), \qquad \rho \geq 0, \tag{5}$$

where F_0 is the baseline labor-market value and H is the additional value generated by worker–firm complementarities in skill formation. The parameter ρ measures the strength of this complementarity channel. This representation is reduced form, but it is consistent with the primitive transition $x'' = h_\rho(x', y)$ when stronger worker–firm complementarity raises the second-period value of high-skill workers in high-productivity firms.

For the main comparison, I focus on a finite-type environment, which is sufficient for the cutoff result and the examples below. Let s^P denote the ordered post-school skill vector induced by school PAM, and s^N the ordered post-school skill vector induced by school NAM. Given labor-market PAM, define

$$V^P(\rho) = \sum_r F_\rho(s^P_{(r)}, y_{(r)}), \qquad V^N(\rho) = \sum_r F_\rho(s^N_{(r)}, y_{(r)}), \tag{6}$$

where $y_{(r)}$ denotes ordered firm productivity. Let

$$\Delta(\rho) = V^P(\rho) - V^N(\rho). \tag{7}$$

Using (5),

$$\Delta(\rho) = \Delta_0 + \rho\Delta_H, \tag{8}$$

where

$$\Delta_0 = \sum_r \left[F_0(s^P_{(r)}, y_{(r)}) - F_0(s^N_{(r)}, y_{(r)})\right], \tag{9}$$

and

$$\Delta_H = \sum_r \left[H(s^P_{(r)}, y_{(r)}) - H(s^N_{(r)}, y_{(r)})\right]. \tag{10}$$

Proposition 3.1 states the cutoff result. The purpose of the cutoff result is not to derive a new sorting theorem, but to isolate the economic force that determines whether the Durlauf-type non-PAM result persists once the labor market becomes a skill-formation environment.

Proposition 3.1 (School PAM under sufficiently strong worker–firm complementarity).

Suppose F_ρ is supermodular in (x', y), so that labor-market matching is PAM. Suppose further that

$$\Delta_0 < 0 < \Delta_H.$$

Then there exists a cutoff

$$\rho^* = -\frac{\Delta_0}{\Delta_H} > 0$$

such that school NAM is optimal for $\rho < \rho^*$, while school PAM is optimal for $\rho > \rho^*$.

Proof. Since F_ρ is supermodular, the labor market matches post-school skills and firm productivities positively assortatively. Therefore, conditional on the school assignment, the planner compares $V^P(\rho)$ and $V^N(\rho)$. School PAM is optimal if and only if

$$\Delta(\rho) = V^P(\rho) - V^N(\rho) \geq 0.$$

By (8),

$$\Delta(\rho) = \Delta_0 + \rho\Delta_H.$$

If $\Delta_0 < 0 < \Delta_H$, then $\Delta(\rho) = 0$ at

$$\rho^* = -\frac{\Delta_0}{\Delta_H} > 0.$$

Hence $\Delta(\rho) < 0$ for $\rho < \rho^*$, so school NAM is optimal, while $\Delta(\rho) > 0$ for $\rho > \rho^*$, so school PAM is optimal. □

Illustration. The cutoff need not be large. Consider the counterexample in Proposition 2.1. School PAM generates

$$s^P = (1, 16),$$

while school NAM generates

$$s^N = (4, 4),$$

and firm productivities are

$$y = (1, 2).$$

Under the baseline fixed-skill payoff

$$F_0(x, y) = \left(2 - \frac{1}{x}\right) y,$$

school NAM dominates school PAM by

$$V^N(0) - V^P(0) = \frac{21}{4} - \frac{39}{8} = \frac{3}{8}.$$

Now let the worker–firm complementarity component be

$$H(x, y) = y^2 x^2.$$

The school-PAM gain from this term relative to school NAM is

$$\Delta_H = \sum_r y_{(r)}^2 \left[(s_{(r)}^P)^2 - (s_{(r)}^N)^2 \right] = 1^2(1^2 - 4^2) + 2^2(16^2 - 4^2) = 945.$$

Thus

$$\rho^* = \frac{3/8}{945} \approx 0.00040.$$

The example is not intended to discipline the magnitude of ρ. Rather, it shows that the threshold can be small in a simple environment where school PAM creates a sufficiently strong upper tail of post-school skills.

3.3 Fixed Worker Skill

The fixed-skill case is the closest analogue to the two-period model in Section 2. In this case,

$$h(x', y) = x'. \tag{11}$$

The composite value becomes

$$F^0(x', y) = f(x', y) + f(x', y) = 2f(x', y).$$

Proposition 3.2 (Fixed worker skill)**.** *Suppose worker skill is fixed during the labor market, so that $h(x', y) = x'$. Then the three-period problem is equivalent to the two-period problem in Section 2, with the labor-market payoff multiplied by two. Hence Proposition 2.1 applies: school PAM need not be optimal.*

Proof. When $h(x', y) = x'$, the planner's objective becomes

$$\int_{X'} 2f(x', \mu(x'))\, d\Gamma'_{\chi}(x').$$

Multiplying the two-period objective in Section 2 by a positive constant does not change the optimal assignment. Therefore the same counterexample applies, and school PAM need not be optimal. □

3.4 Autonomous Worker Skill Evolution

Finally, suppose worker skill changes over time but independently of firm productivity:

$$h(x', y) = \alpha_0(x'), \qquad \alpha_0'(x') > 0. \tag{12}$$

The composite value is

$$F^{\alpha}(x', y) = f(x', y) + f(\alpha_0(x'), y). \tag{13}$$

Proposition 3.3 (Autonomous worker skill evolution)**.** *Suppose worker skill evolves autonomously according to $h(x', y) = \alpha_0(x')$, where α_0 is increasing. If F^{α} is supermodular, labor-market matching is PAM. However, school PAM need not be optimal.*

Proof. If F^{α} is supermodular, the same pairwise-exchange argument used in Proposition 2.1

implies that the labor market matches post-school skill and firm productivity positively assortatively.

It remains to show that school PAM is not generally guaranteed. The fixed-skill case is nested in the autonomous case by setting $\alpha_0(x') = x'$. By Proposition 3.2, there are economies in which school NAM strictly dominates school PAM in that case. Since the planner's value is continuous in α_0, the same strict ranking persists for autonomous skill laws sufficiently close to the identity, for example $\alpha_0(x') = (1+\varepsilon)x'$ with $\varepsilon > 0$ sufficiently small. Thus skill growth by itself does not make school PAM generally optimal; what matters is whether skill formation depends complementarily on firm productivity. □

4 Conclusion

This paper studies how post-school skill formation in the labor market shapes the optimal assignment of students in schools. Building on Durlauf (2026), I show that positive assortative matching in schools need not be efficient once school assignments affect the distribution of skills entering a subsequent labor market. This result continues to hold when worker skill remains fixed during the labor-market stage, and it can also persist when skill evolves autonomously, independently of the matched firm. Thus, the failure of school PAM is not driven by dynamics alone, but by how the school-induced skill distribution is valued in the downstream labor market.

The main result is that worker–firm complementarities in post-school skill formation can change the optimal school assignment. When high-skill workers benefit disproportionately from being matched with high-productivity firms, the upper tail of the school-induced skill distribution becomes more valuable, and school PAM can be optimal under plausible conditions. The analysis therefore suggests that education policy cannot be evaluated solely from the school production function. The efficient assignment of students also depends on how human capital evolves after schooling and on the complementarities between workers and firms in the labor market.

Declarations: Generative AI statement

During the preparation of this work the author used ChatGPT in order to proofread the paper. After using this tool/service, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

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A Proof of Proposition 2.1

Proof. Fix a feasible school assignment χ. The induced post-school skill distribution is

$$\Gamma'_\chi = \big(g(\cdot, \chi(\cdot))\big)_{\#}\Gamma.$$

For any two workers $x'_2 > x'_1$ and two firms $y_2 > y_1$, strict supermodularity of f implies

$$f(x'_2, y_2) + f(x'_1, y_1) > f(x'_2, y_1) + f(x'_1, y_2).$$

Thus any inverted firm assignment can be improved by swapping the two firm types. Repeated pairwise swaps imply that the optimal firm assignment is increasing in post-school skill x'. Hence the labor-market assignment is PAM.

It remains to show that school PAM need not be optimal. Consider a two-type economy with equal masses:

$$x_L = 1, \quad x_H = 4, \qquad z_L = 1, \quad z_H = 4, \qquad y_L = 1, \quad y_H = 2.$$

Let

$$g(x, z) = xz, \qquad f(x', y) = \left(2 - \frac{1}{x'}\right) y.$$

Both functions are strictly increasing on the relevant domain. Moreover, g is strictly supermodular and

$$f_{x'y}(x', y) = \frac{1}{(x')^2} > 0,$$

so f is strictly supermodular.

Under school PAM, terminal skills are

$$g(1, 1) = 1, \qquad g(4, 4) = 16.$$

Since the labor market matches workers and firms positively assortatively, aggregate output is

$$f(1,1) + f(16,2) = 1 + \left(2 - \frac{1}{16}\right) 2 = \frac{39}{8}.$$

Under school NAM, terminal skills are

$$g(1,4) = 4, \qquad g(4,1) = 4.$$

Aggregate output is

$$f(4,1) + f(4,2) = \left(2 - \frac{1}{4}\right)(1+2) = \frac{21}{4}.$$

Since

$$\frac{21}{4} > \frac{39}{8},$$

school NAM strictly dominates school PAM. Therefore, even when both g and f are strictly supermodular, the optimal school assignment need not be positive assortative. □